Thermodynamics of Misaligned Expectations : Work Extraction

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Abstract

Landauer's bound gives the minimum dissipation following erasure through a quantum thermodynamic process. Recent developments in Quantum Thermodynamics and Quantum Information have shown that during a general quantum thermodynamical process, it is impossible to reach exactly Landauer's bound for non-equilibrium processes. Injecting any other input state aside from the minimally dissipative state causes extra dissipation (entropy production). The theoretical foundation of studying extra dissipation has been verified for erasure protocols. The minimally dissipative states are found for two work-extraction protocols that subject a single qubit coupled to a bosonic bath to a time-dependent Hamiltonian. The change in entropy, heat released and work extracted is computed for various initial states and compared.

Introduction

Maxwell's demon is a paradox of apparent energy extraction by a thermodynamic process while violating the Second Law of Thermodynamics. The Szilard engine is a formulation of such a process that involves information erasure. Resolving the apparent violation of the second law of Thermodynamics, Landauer's principle states that the minimum amount of work necessary for information erasure is $k_B T \ln(2)$ where T is temperature and k_B is Boltzmann constant. Additionally, the erasure results into physical entropy production. However, Landauer's principle holds for an initial state that results into the least entropy production. When other initial states are injected, extra dissipation (entropy production) is observed as a result of what can be classically seen as "misaligned expectations". The extra dissipation has been studied for RESET protocols[1]. The project focuses on finding the minimally dissipative initial states for thermodynamic processes which extract work.

Background

Entropy

The following defines information entropy for a general quantum system (known as Von Neumann Entropy) represented by state ρ:

$$S(\rho) = -\mathrm{Tr}\{\rho \log_2(\rho)\}. \tag{1}$$

Landauer's principle can generalised : the amount of work required to erase information represented by the density operator ρ is given by $k_B T S(\rho)\ln(2)$ where $S(\rho)$ is von Neumann Entropy. Hence, we can see that a physical quantity can be linked to information.

Landauer quantified minimum amount of energy required to erase one bit of information, or the entropy production (dissipation) associated with the process. (Note that total (physical) entropy production or dissipation is defined as $\sigma = -Q/T + \Delta S_{sys}$ where Q is the heat absorbed and ΔS_{sys} is the change in entropy of the system). However, a deviation from the expected amount of dissipation occurs when the initial state is not minimally dissipative[2].

Minimum Dissipative Initial State

The minimum dissipative state can be computed for two-level quantum systems.[1]. This sections involves a brief discussion of the method to find the minimally dissipative state.

Note that any (two-dimensional) density operator can be represented as $\rho = \frac{1}{2}(I + +\vec{a} \cdot \vec{\phi})$ where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is just a vector of pauli matrices. Consider four linearly independent input density matrices ρ_0,ρ_1,ρ_2,ρ_3 ; the bloch vectors associated with them $\vec{a}_0$,$\vec{a}_1$,$\vec{a}_2$, $\vec{a}_3$ and the heat or entropy flow associated with them $\mathbf{\Phi_0}$,$\mathbf{\Phi_1}$,$\mathbf{\Phi_2}$, $\mathbf{\Phi_3}$. Define $\vec{\phi}_0$ and $\vec{\phi} = (\phi_x, \phi_y, \phi_z)$ such that

$$A \begin{bmatrix} \phi_0 \\ \phi_x \\ \phi_y \\ \phi_y \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi_0} \\ \mathbf{\Phi_x} \\ \mathbf{\Phi_y} \\ \mathbf{\Phi_y} \end{bmatrix}, \text{ where } A = \begin{bmatrix} 2 & \vec{a}_0 \\ 2 & \vec{a}_1 \\ 2 & \vec{a}_2 \\ 2 & \vec{a}_3 \end{bmatrix} \tag{2}$$

. Now, the Bloch vector of the minimally dissipative state is given by $-\tanh(\phi/2)\hat{\phi}$ where $\phi = |\vec{\phi}|$ and $\hat{\phi} = \vec{\phi}/\phi$.

Quantum Open System: Lindbladian Dynamics

The evolution of a quantum open system can be studied by Lindbladian dynamics. Consider a system coupled to its environment. The total Hamiltonian is given by $H_{tot} = H + H_{env} + H_{int}$ where H , H_{env} and H_{int} are the system, environment and interaction Hamiltonian respectively. In the weak-coupling limit, the master equation governing the system evolution is given by

$$\frac{d\rho}{dt} = [H + H_{LS}, \rho] + \mathcal{D}(A_k) \tag{3}$$

where

$$\begin{aligned} \mathcal{D}(A_k) &= \sum_{\omega} \sum_{k,l} \gamma_{k,l}(\omega)[A_l(\omega)\rho A_k^{\dagger}(\omega) \\ &- \frac{1}{2}\{A_k^{\dagger}(\omega)A_l(\omega), \rho\}] \end{aligned} \tag{4}$$

where H_{LS} is correction to local Hamiltonian, $A_k(\omega)$ are specific system operators in frequency domain, γ_{kl} are coefficients determined by the bath operators [3]. Note that the summation over ω is summation over all possible energy differences between various energy eigenstates.

Methods

A two-level quantum system with a single qbit coupled to a bath of bosons is considered. The Lindblad master equation governing the system is as follows:

$$\dot{\rho}(t) = -i\,[H(t), \rho(t)] + E(t)(N(t)+1)\mathcal{D}[L(t)](\rho(t)) + E(t)N(t)\mathcal{D}[L^{\dagger}(t)](\rho(t)) \tag{5}$$

where

$$\mathcal{D}[L(t)](\rho(t)) = L\rho L^{\dagger} - \frac{1}{2}\left\{LL^{\dagger}, \rho\right\} \tag{6}$$

,and $N(t) = \frac{1}{e^{-\beta E(t)}-1}$. Note that $L(t)$ is just the lowering operator.

For our protocol, the system is subjected to the following time dependent Hamiltonian:

$$H(t) = \frac{1}{2}E(t)(\cos(\theta(t))\sigma_z + \sin(\theta(t))\sigma_x), \tag{7}$$

and the respective time-dependent lowering operator is $L(t) = \frac{1}{2}\left[\cos(\theta(t))\sigma_x - i\sigma_y - \sin(\theta(t))\sigma_z\right]$.

For both the protocols, $E(t) = E(0) - (E(0) - E(\tau))\sin^2(\pi t/(2\tau))$ where $E(0) = E(\tau)/50$. For protocol 1, $\theta(t) = \pi\frac{t}{\tau}$ and for protocol 2, $\theta(t) = \pi$.

Note the definition of the quantities heat (Q), work (W) and entropy produced (Σ).

$$Q(\tau) = \int_0^{\tau} \mathrm{Tr}(\dot{\rho}(t)H(t))dt \tag{8a}$$

$$W(\tau) = \int_0^{\tau} \mathrm{Tr}(\rho(t)\dot{H}(t))dt \tag{8b}$$

$$\Sigma = -Q/T + S(\rho) \tag{8c}$$

where T is Temperature of the baths.

Results

The two work extraction protocols were run on the following inputs:

$$\rho_1 = \begin{bmatrix} 0.5 & 1 \\ 1 & 0.5 \end{bmatrix}, \rho_2 = \begin{bmatrix} 0.25 & 1 \\ 1 & 0.75 \end{bmatrix}, \rho_3 \begin{bmatrix} 0.5 & -0.5i \\ 0.5i & 0.5 \end{bmatrix} \tag{9}$$

along with σ_0 which is the minimally dissipative state for the respective protocols found via the method outlined previously.

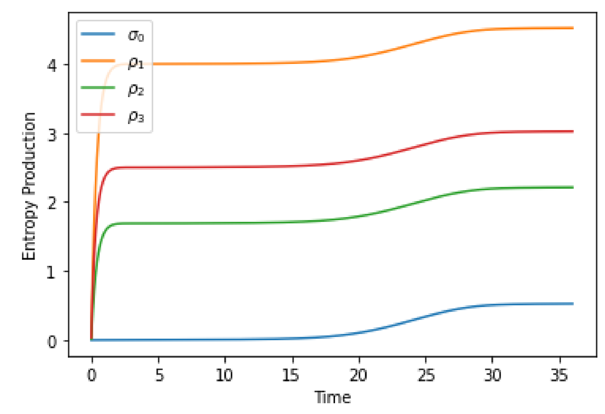

Figure 1: Entropy Produced via Protocol 1

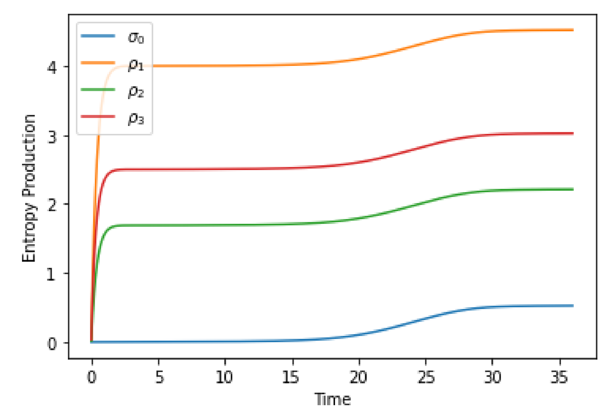

Figure 2: Entropy Produced via Protocol 2

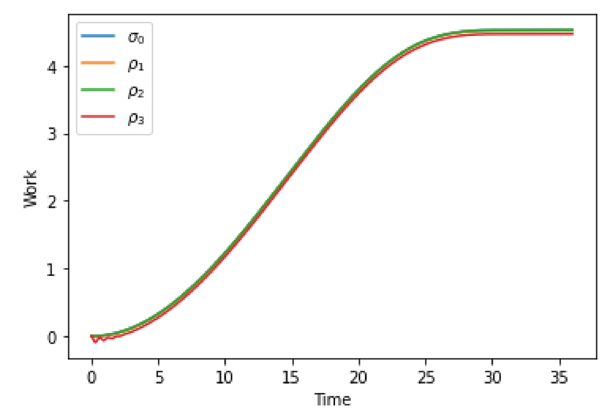

Figure 3: Work Extracted via Protocol 1

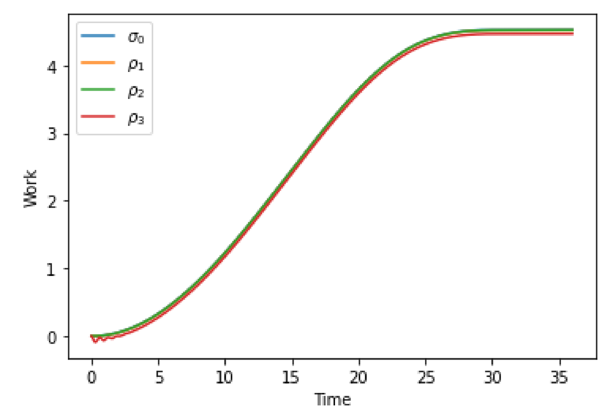

Figure 4: Work Extracted via Protocol 2

Conclusions

- Impossibility of Landauer's bound: It was found that the dissipation came close to Landauer's bound on slowing down the process. However, as it is a non-equilibrium process, the bound is not exactly achieved.
- There is extra dissipation caused by injecting an initial state other than the minimally dissipative state. The minimum dissipative state can be found out by following the procedure outlined before.

Forthcoming Research

The same results could be studied in other more complex systems. There is prospect to study entropy production on injection of an initial state other than the minimally dissipative for the case of a particle in an asymmetric double well potential coupled to a bosonic bath. The double well potential could be used to implement Quantum Szilard Engine.

References

[1] Paul M Riechers and Mile Gu. The impossibility of landauer's bound for almost every quantum state. *arXiv preprint arXiv:2103.02337*, 2021.

[2] Paul M Riechers and Mile Gu. Initial-state dependence of thermodynamic dissipation for any quantum process. *Physical Review E*, 103(4):042145, 2021.

[3] Angel Rivas and Susana F Huelga. *Open quantum systems*, volume 10. Springer, 2012.

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